PYRAMID
IIT-JEE | MEDICAL | FOUNDATION

'लक्ष्य' TEST SERIES 2021-22

STD : XII

SUBJECT : MATHEMATICS

TOPIC : FULL SYLLABUS

TIME : 3 HR. **MAX. MARKS : 80**

General Instructions :

*Question paper consists of **34** questions divided into FOUR sections, namely **A, B, C** and **D**.*

(1) **Section - A :** *Q. No. 1 contains 8 multiple choice type questions carrying two mark each.*

Q. No. 2 contains 4 very short answer type questions carrying one mark each.

(2) **Section - B :** *Q. No. 3 to Q. No. 14 are 12 short answer-I type questions carrying two marks each. Attempt any eight questions.*

(3) **Section - C :** *Q. No. 15 to Q. No. 26 are 12 short answer—11 type questions carrying three marks each. Attempt any eight questions.*

(4) **Section - D :** *Q. No. 27 to Q. No. 34 are 8 long answer type questions carrying four marks each. Attempt any five questions.*

2. *Figures to the right indicate full marks.*
3. *Start each section on new page.*
4. *For each MCQ, the correct answer must be written along with its letter of alphabet : e.g.. (a) ...... / (b) ...... / (c) ...... / (d) ......, etc.*
5. *Evaluation of each MCQ would be done for the first attempt only.*
6. *Use of graph paper is not necessary. Only rough sketch of graph is expected.*
7. *Use log tabe if necessary. Use of calculator is not allowed*

'You should never let your fears prevent you from doing what you know is right.'

Aung San Suu Kyi - Burmese politician, diplomat and author (b.1945)

SECTION - A (20 M)

Q.1 Select and write the most appropriate answer to the given alternatives for each question. (16 M)

i. The quantifier lies in the statement $\sin 2\theta = 2\sin\theta\cos\theta$

(a) Universal (b) Existential (c) Both (d) None

ii. The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

(a) $\left(\frac{-2\pi}{3}\right)$ (b) $\frac{4\pi}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{-\pi}{3}$

iii. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and A(adj A) = KI, then the value of K is

(a) 1 (b) -1 (c) 0 (d) -3

iv. If the line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$, lies in the plane $3x - 14y + 6z + 49 = 0$ then the value of m is

(a) -5 (b) 5 (c) 2 (d) 3

v. $\int \frac{e^x(x-1)}{x^2}\,dx$ _____

(a) $\frac{e^x}{x} + c$ (b) $\frac{e^x}{x^2} + c$ (c) $\left(x - \frac{1}{x}\right)e^x + c$ (d) $xe^{-x} + c$

vi. The particular solution of the D.E. $\frac{dy}{dx} + \frac{y}{x\log x} = 0$, given that y = 2 when x = e is

(a) $y = e^{x/2}$ (b) $y = e^{2/x}$ (c) $y = e^{y/2}$ (d) $y = e^{2/y}$

vii. If $X \sim B(4, P)$ and $P(X=0) = \frac{16}{81}$ then $P(X=4) =$

(a) $\frac{1}{16}$ (b) $\frac{1}{81}$ (c) $\frac{1}{27}$ (d) $\frac{1}{8}$

viii. If $y = a\cos(\log x)$ and $A\dfrac{d^2y}{dx^2} + B\dfrac{dy}{dx} + c = 0$, then the value of A, B, C are ______

(a) $x^2, -x, -y$ (b) x^2, x, y (c) $x^2, x, -y$ (d) $x^2, -x, y$

Q.2 Answer the following **(4 M)**

i. Evaluate $\int (6x+5)^{3/2}\ dx$

ii. Find the area of the region bounded by the curve $y = \sin x$, x-axis and the lines $x = 0$ and $x = \dfrac{\pi}{2}$

iii. Find the joint equation of co-ordinate axes.

iv. Show that the lines

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \text{ and}$$

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + \hat{k}) \text{ are}$$

perpendicular to each other.

SECTION - B (16 M)

Attempt any Eight of the following.

Q.3 Construct the Truth Table for the statement pattern.

$$(p \wedge \sim q) \leftrightarrow (p \rightarrow 2)$$

Q.4 Find the vector equation of the line passing through the point $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpedicular to the vectors. $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$

Q.5 In ΔABC, prove that $a = b\cos c + c\cos B$

Q.6 If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is three times the other, prove that $3h^2 = 4ab$.

Q.7 If $\bar{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\bar{b} = \hat{i} - 2\hat{j} + \hat{k}$ find $\bar{a} \times \bar{b}$.

Q.8 Find the cartesian equation of the plane passing through A(6, 8, 7) and parallel to the plane $\bar{r}.(\hat{i} + \hat{j} + \hat{k}) = 0$

Q.9 The displacement of a particular at time t is given by $S = 2t^3 - 5t^2 + 4t - 3$ find the time when acceleration is 14 ft/sec^2.

Q.10 Evaluate : $\int \frac{1}{\sin x . \cos x} dx$

Q.11 Evaluate : $\int \frac{1}{3x + 7x^{-n}} dx$

Q.12 Prove that $\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$

Q.13 Verify that $y = \log x + c$ is a solution of the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

Q.14 The probability mass function (p.m.f) of x is given below

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Find expected value of x.

SECTION - C (24 M)

Attempt any Eight of the following

Q.15 Show that $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

Q.16 Solve the equation $2[\tan^{-1}(\cos x) = \tan^{-1}[2 \text{ cosec } x]$

Q.17 If the acute angle between the lines $ax^2 + 2hxy + by^2 = 0$ is 60°, then show that $(a+3b)(3a+b) = 4h^2$

Q.18 If l, m, n are the direction cosine of a line, then prove that $l^2 + m^2 + n^2 = 1$

Q.19 If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find k.

Q.20 Prove by vector method, that the angle subtended on semicircle is a right angle.

Q.21 Find $\frac{dy}{dx}$, If $y = x^x + e^x$

Q.22 Find the maximum and minimum value of $f(x)=x^3-3x^2-24x+5$.

Q.23 Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$

Q.24 Find the area of sector bounded by the circle $x^2+y^2=16$

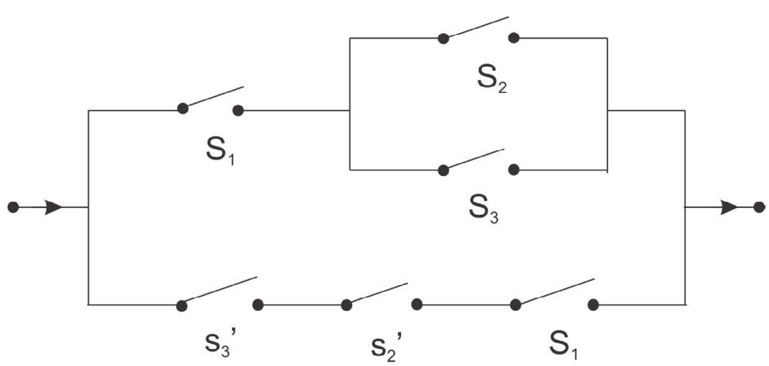
Q.25 Three coins are tossed simultaneously. X is the number of heads. Find expected value and variable of X.

Q.26 In binomial distribution with five Bernoulli's Trials, probability of one and two success are 0.4096 and 0.2048 respectively. Find probability of success.

SECTION - D (20 M)

Attempt any FIVE of the following

Q.27 Give an alternative equivalent simple circuits for the following circuit.



Q.28 If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find A^{-1}.

Q.29 If $\bar{a},\bar{b}$ and $\bar{c}$ are three non-zero, non-coplanar vectors. Prove that any vector $\bar{r}$ in space can be uniquely expressed as a linear combination of $\bar{a},\bar{b},\bar{c}$.

Q.30 Minimize

$z=6x+21y$

subject to

$x+2y\geq 3$

$x+4y\geq 4$

$3x+y\geq 3$

$x\geq 0\quad y\geq 0$

Q.31 If $y = \cos(m\cos^{-1}x)$ then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$

Q.32 The volume of a spherical ball is increasing at the rate of $4\pi\ cc/\sec$ find the rate at which the radius and the surface area are changing when the volume is 288 π cc.

Q.33 Evaluate $\int_{-\pi}^{\pi} \frac{x(1+\sin x)}{1+\cos^2 x}dx$

Q.34 If the population of a country doubles in 60 years, in how many years will it be triple under the assumption that the rate of increase is proportional to the number of inhabitants?
(given log 2 = 0.6912 and log 3 = 1.0986)

❍ ❍ ❍